An environmental engineer is working with a team planning the restoration of a section of stream that historically supported a healthy population of Rainbow trout. Over a period of years, stream water temperature increased, and dissolved oxygen levels decreased, reducing the population of the native Rainbow trout. In addition to water condition changes, someone had introduced German Brown trout into the stream. The Brown trout are more aggressive than the native Rainbows, and during a period of low Rainbow population, were able to out-compete with the Rainbows for food. The net result of increased competition for food, and a reduction in stream habitat quality, Rainbow trout populations in the stream had been reduced to zero.

As part of the reintroduction plan, a model of the population dynamics of the native Rainbow and introduced Brown trout was developed. The model uses typical competing population growth equations

\[
\frac{dN_1}{dt} = (\alpha_1 - \beta_1 N_1 - \gamma_1 N_2)N_1
\]

\[
\frac{dN_2}{dt} = (\alpha_2 - \beta_2 N_1 - \gamma_2 N_2)N_2
\]

where \(N_1\) and \(N_2\) is the density of Rainbow and Brown trout (in fish/100m) respectively, \(t\) is time in years, and \(\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2\) are model constant coefficients. Using data from similar streams, values of the model coefficients are assumed as follows: \(\alpha_1 = 1/\text{year}, \alpha_2 = 0.75/\text{year}, \beta_1 = 0.007/\text{Rainbow-year}, \beta_2 = 0.007/\text{Rainbow-year}, \gamma_1 = 0.01/\text{Brown-year}, \gamma_2 = 0.007/\text{Brown-year}.

Prior to replanting the stream with Rainbow trout, the plan is to reduce (by netting) the Brown trout population to 25 fish/100m of stream length. Rainbow trout fry will then be planted at a density of 4 fish/100m of stream length. Stable populations of both species is expected within 10 years.

Determine whether the management plan, as proposed will produce a stable (and viable) population of Rainbow trout with a 10 year period. Verify that the population will remain reasonably stable for 50 years. If the Brown trout overwhelm the Rainbow trout, determine by trial and error the maximum Brown trout population required to produce a stable Rainbow population over a 50 year period. Finally, check on the stability of the population of both species over a 350 year period.