Engineering problems frequently require the solution of systems of equations, where the equations may be algebraic, differential, or integral. The system of equations will usually be represented in matrix form, and the solution of these equations will commonly involve basic matrix operations such as matrix multiplication.

Write a FORTRAN program that allows a user to enter two matrices up to size 5 X 5. The user should first enter the size of the first matrix, and then the matrix itself. Then the user should enter the size of the second matrix and the matrix itself (be sure to provide suitable prompts). Your program should check to make sure that matrix multiplication is defined (i.e. the inner dimensions are equal) for the matrices provided and print out an error message if it is not. The program should print out both of the matrices that were provided and their product in a neat matrix form. Calculate the product in two ways, using the intrinsic function `matmul` and using your own subroutine that computes the matrix product with do-loops. Use allocatable arrays to match the size of the problem that the user defines.

To test out your program, find $AB$ and $BA$ given

\[
A = \begin{pmatrix}
1 & 0 & 6 \\
2 & -3 & 1
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]