There are many instances where an engineer needs to integrate a function over some specified interval. While there are a number of different schemes that can be used to numerically integrate a continuous function, one algorithm that is easy to implement is called the trapezoid rule method. This algorithm has the following steps for integrating a function $f(x)$ over the limits $x_L$ to $x_U$.

1. Specify the number of equal width intervals ($n$) to use between $x_L$ and $x_U$. Up to a point, increasing the number of intervals improves the accuracy of the solution but increases the execution time.
2. Compute the width of each interval

$$h = \frac{x_U - x_L}{n}$$

3. Compute the desired integral using the equation

$$\int_{x_L}^{x_U} f(x) \, dx \approx h \left[ \frac{f(x_L)}{2} + \sum_{i=1}^{n-1} f(x_L + i \cdot h) + \frac{f(x_U)}{2} \right]$$

Write a program that implements the trapezoid rule integration algorithm. The program should ask the user for the lower and upper bound of integration and the number of intervals. The program should then compute and write out the integration results. The function that is to be integrated should be contained in an external function subprogram. Use formatted output statements for any prompts (including advance="no") and for the results. Use floating point variables with at least 14 significant digits. Test your program by integrating the function

$$f(x) = 6x^3$$

between the limits of -1 and 2, using 50 intervals (trapezoids), then use your program to solve the problem below.
If the velocity distribution of a fluid flowing through a pipe is known, the flow rate $Q$ (the volume of water passing through the pipe per unit time) can be computed by

$$Q = \int v \, dA$$

where $v$ is the fluid velocity and $A$ is the pipe's cross-sectional area. For a circular pipe,

$$A = \pi r^2$$

and

$$dA = 2\pi r \, dr$$

Therefore the flowrate passing through a circular area of radius $r_d$ is

$$Q = \int_0^{r_d} v(2\pi r) \, dr$$

where $r$ is the radial distance measured outward from the center of the pipe. If the total flowrate passing through a pipe is to be determined, then $r_d$ is just the radius of the pipe, $r_p$. From measurements on a 6 inch diameter irrigation pipe, the velocity profile (in ft/sec) was found to be

$$v = 3 \left( 1 - \frac{r}{r_p} \right)^{1/7}$$

where $r_p$ is 3 inches.

After testing, use the program to determine the flowrate of the irrigation pipe for the velocity profile given above. Investigate the effect of changing the number of intervals used in the numerical approximation.