1. Work problem 48 in chapter 2, section 3, using both Gauss elimination and LU decomposition.

2. Consider the system of linear equations $\mathbf{Ax} = \mathbf{b}$ where

\[
\begin{pmatrix}
1 & 1/2 & 1/3 & 1/4 \\
1/2 & 1/3 & 1/4 & 1/5 \\
1/3 & 1/4 & 1/5 & 1/6 \\
1/4 & 1/5 & 1/6 & 1/7
\end{pmatrix}
\begin{pmatrix}
p_7 \\
p_8 \\
p_9 \\
p_{12}
\end{pmatrix}
= 
\begin{pmatrix}
77/12 \\
37/10 \\
53/20 \\
218/105
\end{pmatrix}
\]

which has solution $\mathbf{x}^T = [4 \ 3 \ 2 \ 1]$. Compute the condition number of $\mathbf{A}$ and explore the sensitivity of the solution to changes in coefficients in $\mathbf{A}$. For example, determine the solution vector $\mathbf{x}$ when $a_{4,1} = 0.26$.

3. The state water commission is considering a permit for a new ground water well in a small confined aquifer. A cross-section of the aquifer is shown in Figure 1 below. At a 1600 meter size (into the paper), the other two aquifer boundaries can be described as no-flow boundaries and the proposed well site is in the center of the 1200 meter by 1600 meter area (see Figure 2). Several small ranch houses are in the area of the proposed well, and these houses have existing small well systems. These ranches are concerned that their wells will go dry if the new pump is approved.

As a first step in determining the impact of the proposed well, a numerical model of the steady-state groundwater potential field is developed using finite difference approximations of the resulting partial differential equations. The partial differential equations that describe the 2-dimensional steady-state relationship between the groundwater head and position in a confined, homogeneous, isotropic aquifer is

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0
\]

Using a simple computational grid with square elements, the finite difference approximations to the governing partial differential equation result in the following system of equations for the pressure head at each of the 6 computational nodes shown in Figure 3.

\[
\begin{align*}
p_7 - 4p_8 + p_9 + p_{13} &= -50 \\
-4p_7 + p_8 + p_{12} &= -95 \\
p_7 - 4p_{12} + p_{13} &= -90 \\
p_8 + p_{12} - 4p_{13} + p_{14} &= -45 \\
p_8 - 4p_9 + p_{14} &= -95 \\
p_9 + p_{13} - 4p_{14} &= -90
\end{align*}
\]

Using Scilab, determine the current steady-state groundwater pressure head field. Plot the groundwater pressure field and contour map using Scilab’s `plot3d1` and `contour` command.
Confined Aquifer $T=200 \text{ m}^2/\text{day}$

Figure 1. Crosssection of confined aquifer system.

Constant head = 50 m

1200 m x 1600 m

Proposed well site

Figure 2. Aerial view of basin.

Figure 3. Computational grid.

This was typeset with TeX.